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Conflict of Interest Wind Modeling in Aircraft Response Study

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The concept of viewing the wind affecting an aircraft as being controlled by an intelligent adversary acting through a wind controller is introduced. The wind controller has objectives which are opposed to those of the aircraft controller. This conflict of interest concept is developed within the framework of linear quadratic differential game theory and applied in a number of related formulations to generate deterministic worst-case wind models for a typical STOL aircraft on the landing approach. The results indicate that the differential games technique encompasses several worst-case concepts and may be tailored to suit a variety of applications, including the generation of wind inputs that attempt to track a specified state trajectory and closed-loop wind controller models for use in aircraft training simulators.

Nomenclature

d	= glidepath deviation normal to glidepath plane, positive above, m
h	= altitude above ground level (AGL), m
I	= identity matrix
J	= payoff functional
q	= pitch rate, rad/s
u	= airspeed component along x stability axis, m s^{-1}
V_e	= reference equilibrium airspeed
W_1	= tailwind velocity, m s^{-1}
W_3	= downdraft velocity, m s^{-1}
W_{1_e}	= reference equilibrium value of W_1, m s^{-1}
w	= airspeed component along z stability axis, m s^{-1}
$\boldsymbol{w}$	= total wind minus the linear mean wind $\boldsymbol{W}_L$
w_1	= see Eq. (13a)
x_1	= horizontal position
γ_G	= glide slope angle, deg
$\Delta\boldsymbol{w}$	= $[\Delta w_1 W_3]^T$
δ_E	= elevator angle, rad
δ_T	= throttle position (fraction of full throttle)
θ	= Euler pitch angle, rad

Notation Conventions

X	= matrix
X^{-1}	= inverse of X
X^T	= transpose of X
$\hat{X}(\hat{x})$	= augmented matrix (vector)
x_0	= initial value of x
x^*	= optimal value of x
$\boldsymbol{x}$	= vector or column matrix
$\lVert\boldsymbol{x}\rVert_A$	= $\sqrt{\boldsymbol{x}^T A \boldsymbol{x}}$
$\Delta(\ )$	= perturbation quantity about a reference equilibrium value

I. Introduction

A NUMBER of recent landing approach and takeoff accidents for which variable winds have been found to be major contributing factors (e.g., the well-documented JFK accident in 1975[1]) have focused attention on wind modeling for hazard definition.[2] While one may proceed with the analysis of many facets of this problem using stochastic techniques, certain aspects are best treated using deterministic models which represent one realization of the wind field. This is true when considering flight through the atmospheric boundary layer where the turbulence properties are functions of at least height, and wind probability models are not well established,[3] and is especially true when considering the effects of winds that lie in the tails of the probability curves whose contribution to the expectation values of the aircraft response may be small, but which nevertheless may create significant safety problems when they do occur. Deterministic models are also required in training simulators.

One may generate such deterministic wind models using a number of techniques and data sources, including the following important ones:

1) Models based on meteorological data,[4] including data obtained from atmospheric boundary-layer wind tunnels.

2) Situation specific dynamic models of the physics of the atmospheric flow (e.g., thunderstorm outflow models[5]).

3) Arbitrary profiles intended to stimulate aircraft dynamic response (e.g., discontinuity profiles).[6]

4) Analytically maximizing worst-case wind models.

The latter group of methods are of particular interest because they look for wind models which maximize a functional (say, J) of the state of the aircraft, and thus pose the worst-case concept being used within a formal mathematical framework. These methods also avoid the difficult task of analytically and computationally modeling complex atmospheric flows.

Such techniques may prove to be quite useful and appear to be relatively unexplored. The only attempt of which the authors are aware at analytically generating worst-case winds *without* specifying the form of the wind vector *a priori* are the efforts of van der Vaart.[3] Van der Vaart considers the maximization (minimization) of the final value of the ith state variable due to the kth disturbance input of a time-invariant, linear system with zero initial conditions, and finds the worst-case input to be given in terms of the time-reversed impulsive response function of this input-output pair for the case where the time integral of the square of the disturbance input is constrained to take on a particular value. Attempts at determining the values of the parameters of a given form of the wind inputs so that certain undesirable characteristics exceed safe values are only slightly more common. Corbin[7] searches for combinations of sines and cosines which produce the worst-case turbulence time histories for the flare and touchdown case. Jones[8] considers ramplike discrete gusts and

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looks for combinations of these gusts that maximize some part of the aircraft response (e.g., normal acceleration).

It is the purpose of this paper to indicate that these techniques are encompassed by the conceptual paradigm of viewing the wind inputs into the aircraft dynamic system as being controlled by an intelligence whose objectives oppose safe aircraft response. In the most general scenario we may imagine a conflict of interest problem where the aircraft controller and the wind are conceived as intelligent adversaries trying to achieve opposing objectives. In such a model the wind may be assumed to have information on the state of the aircraft on which it can base worst-case inputs; i.e., the wind may be conceived to be a feedback controller. This is tantamount to *closing the loop* on the wind.

A technique which provides the analytical tools for handling such two-sided conflict of interest problems for cases where the form of the wind inputs is not specified *a priori* is encompassed by the theory of differential games (DG). In particular, the application of two-player, zero-sum, linear quadratic DG theory to the generation of worst-case wind models for aircraft on the landing approach will be considered. Through several numerical examples an assessment is made of the usefulness of the method and further applications and extensions are anticipated. The theory that will be briefly summarized in the following sections and the numerical examples of Sec. IV are discussed in detail in Ref. 9.

II. Differential Game Theory Background

Although Rufus Isaacs first considered DG in the 1950s, they did not become well known in the control field until after the publication of his text on the subject in 1965.[10] Their application to aerospace systems is relatively uncommon and has generally been limited to conflict of interest controller design rather than as a method of *generating* worst-case disturbances. In particular, a class of pursuit-evasion differential games modeling certain combat situations has received considerable attention.[11]

The formal theoretical treatment of DG (see, for example, Friedman[12]) is quite abstract and is not germane to the engineering applications considered in this paper. Suffice it to say that linear quadratic DG, which we consider here, are amenable to analysis and have a well-developed theoretical framework.

We consider DG played in the time interval $[t_0,t_f]$ on the well-behaved, possibly time-varying linear system

$$\dot{x}(t) = F(t)x(t) + G_1(t)u(t) + G_2(t)\eta(t) + f(t) \tag{1a}$$

$$x(t_0) = x_0 \tag{1b}$$

The quadratic payoff functional is given by

$$J = \left\| x(t_f) - x_d(t_f) \right\|_S^2 + \int_{t_0}^{t_f} \left[\left\| x - x_d \right\|_{Q(t)}^2 + \left\| u \right\|_{R_1(t)}^2 + \left\| \eta \right\|_{R_2(t)}^2 \right] \mathrm{d}t \tag{2a}$$

where $x_d(t)$ is a specified desired state time trajectory, and S, $Q(t)$, $R_1(t)$, and $R_2(t)$ are symmetric weighting matrices required to have the following sign definiteness properties on $[t_0,t_f]$:

$$S, Q(t) \quad \text{are sign indefinite}$$

$$R_1(t) > 0 \tag{2b}$$

$$R_2(t) < 0$$

The weighting matrices are chosen in this way to make the problem meaningful in a conflict of interest scenario. In this formulation player 1 with input vector u (the aircraft controller) attempts to minimize J, whereas player 2 with input vector η (the wind controller) attempts to maximize J. Because the payoff functional is the same for the maximizing and minimizing player, the game is also zero sum.

Using either a generalized Hamilton-Jacobi-Bellman equation technique or a variational calculus technique, one may show that the minimax *feedback* strategies of the two players are given by[9,13,14]

$$u^* = -R_1^{-1} G_1^T [Px + b] \tag{3a}$$

$$\eta^* = -R_2^{-1} G_2^T [Px + b] \tag{3b}$$

where

$$\dot{P} = -PF - F^T P + P[G_1 R_1^{-1} G_1^T + G_2 R_2^{-1} G_2^T] P - Q \tag{4a}$$

$$\dot{b} = -F^T b + P[G_1 R_1^{-1} G_1^T + G_2 R_2^{-1} G_2^T] b + Q x_d - Pf \tag{4b}$$

with terminal conditions

$$P(t_f) = S \tag{5a}$$

$$b(t_f) = -S x_d(t_f) \tag{5b}$$

The equilibrium value of the payoff functional is given by

$$J^*[x(t_0), t_0] = x^T(t_0) P(t_0) x(t_0) + 2x^T(t_0) b(t_0) + c(t_0) \tag{6}$$

where

$$\dot{c} = b^T [G_1 R_1^{-1} G_1^T + G_2 R_2^{-1} G_2^T] b - x_d^T Q x_d - 2b^T f \tag{7}$$

with terminal condition

$$c(t_f) = x_d^T(t_f) S x_d(t_f) \tag{8}$$

The controller inputs (3a) and (3b) are given in feedback form. This has the particular advantage that if the aircraft controller deviates from the minimax strategy, the feedback wind controller will immediately take advantage of this and produce a greater value of J (and vice versa). This is an immediate consequence of the definition of the minimax solution,

$$J(u^*, \eta) \le J(u^*, \eta^*) \le J(u, \eta^*) \tag{9}$$

in combination with the closed-loop form of Eqs. (3a) and (3b).

These results are very similar to the optimal linear quadratic tracking problem solution, with the extra terms in the differential equations (4a), (4b), and (7) arising because of the two-sided nature of the optimization. Equation (4a) is the matrix Riccati equation for this problem.

In contrast to linear quadratic optimal control problems, the solution to conflict of interest formulations may not exist because of the presence of a conjugate point‡ in the interval $[t_0,t_f]$. This conjugate point, however, may be removed from this interval by making the maximizing player's control weightings stronger (i.e., by multiplying R_2 by a large enough positive constant[16]). This is equivalent to constraining the integral on $[t_0,t_f]$ of the weighted Euclidean norm squared of

‡By conjugate points we mean two points A and A' along the optimal trajectory from which the value of the differential game $\{J^*[x(t),t]\}$ is the same.[15] In the linear quadratic conflict of interest problem, conjugate points manifest themselves via a finite-escape time for the matrix Riccati equation (4a). This leads to infinite control inputs (and thus infinite control energies), clearly not a physically acceptable result.

the maximizing player's inputs, i.e., constraining the integral of the "energy" input of the maximizing player. Physically this is a very reasonable constraint.

III. Landing Approach Formulation

The landing approach longitudinal dynamics in variable winds are modeled using perturbation equations written in stability axes. They are linearized about a reference equilibrium consisting of flight on a rectilinear glide slope in the presence of a constant headwind. They take the matrix form

$$A_1\Delta\dot{x}=A_2\Delta x+B_1\Delta\delta+B_2\Delta W+B_3\Delta\dot{W} \tag{10}$$

Here Δx is the perturbation state vector as given by

$$\Delta x=[\Delta u w q \Delta\theta \Delta x_I \Delta h d]^T \tag{11a}$$

$\Delta\delta$ is the perturbation control vector as given by

$$\Delta\delta=[\Delta\delta_E \Delta\delta_T]^T \tag{11b}$$

and ΔW is the perturbation longitudinal wind velocity vector (expressed in inertial coordinates) acting at the center of mass of the aircraft as given by

$$\Delta W=[\Delta W_1 W_3]^T \tag{11c}$$

The wind velocity is assumed to be uniform over the length and span of the aircraft.

Equation (10) was modified to include a linear mean wind profile based on the linearization reference equilibrium altitude h_e of the aircraft. The wind inputs generated by the wind controller will be superimposed onto this linear wind profile in some of the numerical examples. The linear mean wind is given by

$$W_L=\begin{bmatrix} W_{L_{1_0}}+\kappa h_e \\ 0 \end{bmatrix}=\begin{bmatrix} W_{L_1} \\ 0 \end{bmatrix} \tag{12a}$$

$$h_e=\sin(\theta_e)V_e t+h_0 \tag{12b}$$

Including Eqs. (12a) and (12b) into Eq. (10) amounts to making the substitution

$$\Delta W=W_L+w-W_e=\begin{bmatrix} W_{L_1}+w_1-W_{1_e} \\ W_3 \end{bmatrix} \tag{13a}$$

or

$$\Delta W=\begin{bmatrix} W_{L_1}+\Delta w_1 \\ W_3 \end{bmatrix}=W_L+\Delta w \tag{13b}$$

Here Δw_1 represents the perturbation value of the W_1 wind component superimposed over the linear mean wind W_{L_1}. This is equivalent to introducing a time-varying column vector $b_1(t)$ on the right-hand side of Eq. (10), a term analogous to $f(t)$ in Eq. (1a).[9] Thus Eq. (10) becomes

$$A_1\Delta\dot{x}=A_2\Delta x+B_1\Delta\delta+B_2\Delta w+B_3\Delta\dot{w}+b_1(t) \tag{14a}$$

where

$$b_1(t)=B_2W_{L_1}+B_3\dot{W}_{L_1} \tag{14b}$$

Equation (14a) is not in a form analogous to Eq. (1a) because of the presence of both Δw and $\Delta\dot{w}$ terms. This difficulty may be overcome by augmenting the state vector with Δw and treating the disturbance vector $\Delta\dot{w}$ to be the maximizing player's control inputs (this also puts the problem in a form in which Δw may be weighted in the payoff functional). Similarly, to permit weighting of control rate in the payoff functional (for smoothness of control response) and to allow formulations where the wind controller strategy includes feedback based on the aircraft control position vector $\Delta\delta$, the state vector is also augmented with $\Delta\delta$ and the minimizing player's inputs are control rate $\Delta\dot{\delta}$ [analogous to u in Eq. (1a)]. The resulting augmented equations of motion may thus be written

$$\begin{bmatrix} A_1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\Delta\dot{\hat{x}}=\begin{bmatrix} A_2 & B_1 & B_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\Delta\hat{x}$$
$$+\begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}\Delta\dot{\delta}+\begin{bmatrix} B_3 \\ 0 \\ I \end{bmatrix}\Delta\dot{w}+\begin{bmatrix} B_2W_L+B_3\dot{W}_L \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

or

$$\Delta\dot{\hat{x}}=\hat{A}\Delta\hat{x}+\hat{C}_1\Delta\dot{\delta}+\hat{C}_3\Delta\dot{w}+\hat{c}_1(t) \tag{16}$$

where

$$\Delta\hat{x}=[\Delta x^T \Delta\delta^T \Delta w^T]^T \tag{17a}$$

$$(AC_1C_3c_1)=A_1^{-1}(A_2B_1B_3b_1) \tag{17b}$$

The remaining details are left to Ref. 9.

IV. Numerical Examples and Discussion

The intelligent adversary concept is summarized in the feedback block diagram of Fig. 1. A number of formulations are possible as determined by the position of switches A, B, and C. Examples are given for all of these formulations in Ref. 9.

A. Indirect (Perversity Function) Method

For these formulations switch A is closed and switches B and C are open. The wind inputs are treated as control inputs which attempt to make the aircraft track a specified state trajectory which is known to be particularly *perverse*. Such state trajectories may be based, for example, on recorded trajectories in aircraft incidents and accidents. The payoff functional that is to be *minimized* by the wind inputs is given by

$$J=\left\|\Delta x_{\rm op}(t_f)-\Delta x_{{\rm op}_d}(t_f)\right\|_S^2+\int_0^{t_f}\left[\left\|\Delta x_{\rm op}(t)-\Delta x_{{\rm op}_d}(t)\right\|_Q^2\right.$$
$$\left.+\mu\left\|\Delta\dot{w}_{\rm op}\right\|_{R_{21}}^2\right]{\rm d}t \tag{18}$$

$$S,Q\geq 0 \qquad R_{21}>0 \tag{19}$$

The payoff functional is written in terms of $\Delta x_{\rm op}$, $\Delta w_{\rm op}$ rather than Δx, Δw in order to indicate that the optimization need not be carried out with the full dynamic system (15); i.e., $\Delta x_{\rm op}$ and $\Delta w_{\rm op}$ may be a suitably chosen subset of Δx and Δw. This choice of the dynamic system to be optimized must, of course, be a subset of Eq. (15) that is uncoupled from the state variables omitted, e.g., $\Delta x_{\rm op}=[\Delta u, w, q, \Delta\theta]^T$. The positive

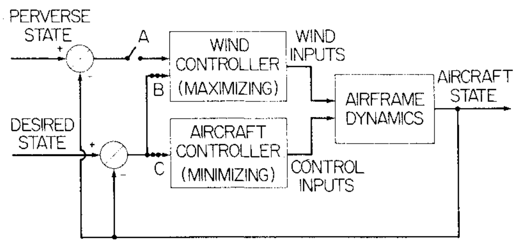

Fig. 1 Conflict of interest wind and aircraft controller modeling: the intelligent adversary concept.

parameter μ may be varied so that a suitable value of the integral

$$S_{ws} = \int_0^{t_f} \Delta \dot{w}_{\mathrm{op}}^T(t) \Delta \dot{w}_{\mathrm{op}}(t)\,\mathrm{d}t \tag{20}$$

is obtained.

B. Direct Method

For these formulations switch B is closed and switches A and C are open. The optimization problem is to find a wind controller that *maximizes* the aircraft deviation from a desired state trajectory $\Delta x_{\mathrm{op}_d}(t)$ in the sense of the quadratic payoff functional

$$J = \left\| \Delta x_{\mathrm{op}}(t_f) - \Delta x_{\mathrm{op}_d}(t_f) \right\|_S^2 + \int_0^{t_f} \left[\left\| \Delta x_{\mathrm{op}}(t) - \Delta x_{\mathrm{op}_d}(t) \right\|_Q^2 + \mu \left\| \Delta \dot{w}_{\mathrm{op}}(t) \right\|_{R_{21}}^2 \right] \mathrm{d}t \tag{21}$$

$$R_{21} < 0 \tag{22}$$

C. Differential Game Method

For these formulations switches B and C are closed and switch A is open. The objective is to find a best-case aircraft controller and a worst-case wind controller that minimaximize the payoff functional

$$J = \left\| \Delta x_{\mathrm{op}}(t_f) - \Delta x_{\mathrm{op}_d}(t_f) \right\|_S^2 + \int_0^{t_f} \left[\left\| \Delta x_{\mathrm{op}}(t) - \Delta x_{\mathrm{op}_d}(t) \right\|_Q^2 + \left\| \Delta \dot{\delta} \right\|_{R_{11}}^2 + \mu \left\| \Delta \dot{w}_{\mathrm{op}} \right\|_{R_{21}}^2 \right] \mathrm{d}t \tag{23}$$

$$R_{11} > 0 \qquad R_{21} < 0 \tag{24}$$

The indirect and direct method formulations discussed above did not include a feedback aircraft controller. This was done for the sake of simplicity. Such an aircraft controller may be readily incorporated in these formulations by defining it *a priori* and including it in the airframe dynamics.

All of the numerical examples to follow are for a typical twin-engine light STOL transport in the landing configuration (landing gear down, full flaps). The aircraft's mass and geometric characteristics are as follows (Ref. 9 gives more details and also summarizes the aerodynamic and natural mode characteristics):

$$m = 4990\ \mathrm{kg} \quad (341.9\ \mathrm{slugs}) \tag{25a}$$

$$S = 39.0\ \mathrm{m}^2 \quad (420\ \mathrm{ft}^2) \tag{25b}$$

$$\bar{c} = 2.0\ \mathrm{m} \quad (6.5\ \mathrm{ft}) \tag{25c}$$

$$b = 20\ \mathrm{m} \quad (65\ \mathrm{ft}) \tag{25d}$$

The simulation conditions are

$$V_e = 40\ \mathrm{m\,s^{-1}} \tag{26a}$$

$$h_0 = 100\ \mathrm{m} \tag{26b}$$

$$\gamma_G = 7\ \mathrm{deg} \tag{26c}$$

$$W_{1_e} = 0 \tag{26d}$$

From Eq. (26d) it follows that

$$\Delta w = [w_1 W_3]^T \tag{27}$$

The numerical results were obtained using computer codes written in IBM Fortran (see Ref. 9 for a description of the algorithms) and run on an IBM 3033 computer.

Table 1 Summary of direct method cases

Case	μ	$w_1(t_f)$ Terminal weighting
1	10.6	0
2	10.9	0
3	11.2	0
4	12.0	0
5	16.0	0
6	10.6	−1
7	11.3	−1

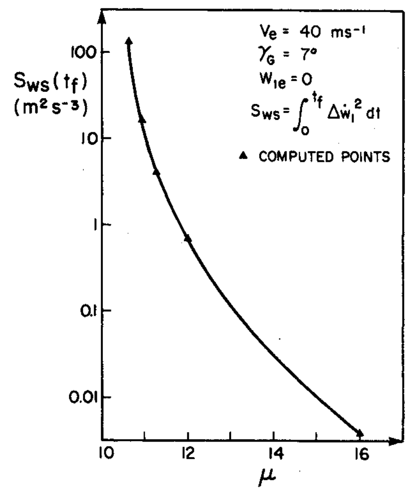

Fig. 2 Variation of $S_{ws}(t_f)$ with μ for direct method cases [no $w_1(t_f)$ weighting].

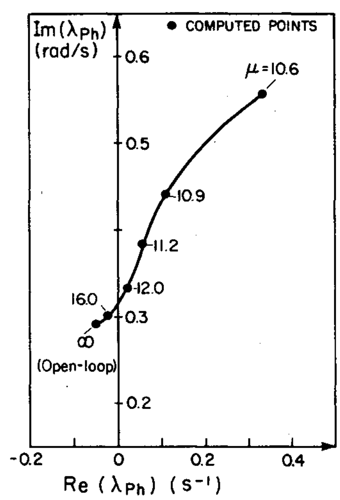

Fig. 3 Phugoid mode root locus with maximizing wind controller for direct method cases [no $w_1(t_f)$ weighting].

The selection of the weighting matrices in the payoff functionals (18), (21), and (23) is arbitrary, although weighting schemes may be used to yield suitable starting values about which to iterate in order to find a satisfactory solution. All of the weighting matrices are restricted to be symmetric and for the purposes of the numerical examples were diagonal. A detailed discussion of the methodology used in selecting the weighting matrices and of the values used in the numerical examples is given in Ref. 9.

Direct Method Example

In this example a wind controller is found that maximizes Eq. (21) under the following conditions:

$$\Delta x_{\mathrm{op}} = [\Delta x_I^T \Delta w_{\mathrm{op}}^T]^T \tag{28a}$$

$$\Delta x_l = [\Delta u w q \Delta\theta]^T \tag{28b}$$

$$\Delta w_{op} = [w_l] \tag{28c}$$

$$W_3 = 0 \tag{28d}$$

$$\Delta x_l(0) = \Delta x_{l_{trim}} + \Delta x_{l_{\Delta trim}} \tag{28e}$$

$$\Delta x_{l_{trim}} = [0, -0.125 \text{ m s}^{-1}, 0, 0.0259 \text{ rad}] \tag{28f}$$

$$\Delta x_{l_{\Delta trim}} = [-2 \text{ m s}^{-1}, 0, 0, 0]^T \tag{28g}$$

$$\Delta w_{op}(0) = [-5 \text{ m s}^{-1}] \tag{28h}$$

$$\dot{W}_3 = 0 \tag{28i}$$

$$W_{L_l} = -0.035\, h_e \tag{28j}$$

$$\Delta\delta(0) = [-8.57\times10^{-4}\text{rad}, 0.0557]^T \tag{28k}$$

$$\Delta\dot{\delta} = 0 \text{ (controls fixed)} \tag{28l}$$

This example amounts to maximizing the dynamic subset of the full equations of motion (16) with w_l inputs superimposed over the linear wind defined by Eq. (28j). The initial conditions consist of trim conditions $\Delta x_{l_{trim}}$ which were determined on the assumption of trimmed flight on a 7-deg glide slope in the wind that existed at the starting height h_0 [i.e., $W_l(h_0) = W_{L_l}(h_0) + w_l(0)$] plus a component $\Delta x_{l_{\Delta trim}}$ representing an initial out of trim perturbation required to excite the system. The controls are fixed at the trim values.

Five cases were run for different values of the parameter μ and for *zero* weighting in the payoff functional of $w_l(t)$ and $w_l(t_f)$ [recall that w_l is part of the state vector and may thus be weighted through S and Q in Eq. (21)]. These cases are summarized in Table 1.

This formulation was found to contain a conjugate point for $\mu < \mu_{cp}$, where μ_{cp} was determined to be bounded by $10.4 < \mu_{cp} < 10.6$. The wind variability, as measured by $S_{ws}(t_f)$ was found to increase dramatically as μ approached its conjugate point value. This may be clearly seen in Fig. 2, where it is also noted that near the conjugate point small changes in μ result in large changes in $S_{ws}(t_f)$. This sensitivity makes it more difficult to determine a μ that yields a particular value of $S_{ws}(t_f)$, i.e., a solution that satisfies a hard "energy" constraint on $\dot{w}_l(t)$.

The worst-case wind inputs that resulted for these cases were found to stimulate the phugoid mode of the aircraft. This is not surprising in view of the much greater damping of the short-period mode. For μ near the conjugate point value, the destabilization of the phugoid mode is substantial. This may be seen in Fig. 3 where the wind control law corresponding to $t=0$ is incorporated in the equations of motion and the resulting phugoid mode eigenvalues are plotted with μ as a parameter.

The worst-case wind profiles, presented with h_e as an altitude coordinate, for cases 3 and 4 of Table 1 are given in Figs. 4 and 5, respectively. These are seen to have substantially nonzero W_l values for $h_e = 0$, a physically unrealistic characteristic for atmospheric boundary-layer wind models. This situation may be improved upon by weighting the terminal value of $w_l(t_f)$, i.e., not making it cost effective for the worst-case solution to have substantially nonzero values at $t = t_f$. This was done for cases 6 and 7 of Table 1, and the resulting wind profiles are given in Figs. 4 and 5, respectively, and compared to cases 3 and 4. It can be seen that the two wind profiles for the terminally weighted cases achieve $w_l(t_f)$ values near zero.

In general, finding a worst-case solution with a desired $S_{ws}(t_f)$ value and terminal wind velocity requires simultaneously varying the constant μ and the terminal

Table 2 Differential game example cases

Case	μ	J^*
8	0.7584	0.04774
9	0.7424	0.8552

Table 3 Phugoid mode eigenvalues for infinite terminal time minimax solution of differential game example

Particulars	
Open loop	$-0.0521 \pm i0.293$
Aircraft controller alone	$-0.423 \pm i0.346$
Wind controller alone	$0.109 \pm i0.302$
Minimax roots	$-0.262 \pm i0.272$

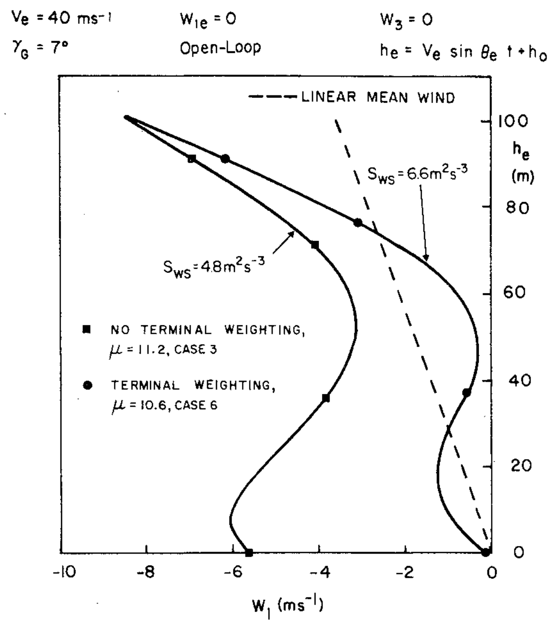

Fig. 4 Direct method example wind profiles. Effect of terminal weighting on $w_l(t_f)$ for large values of S_{ws}.

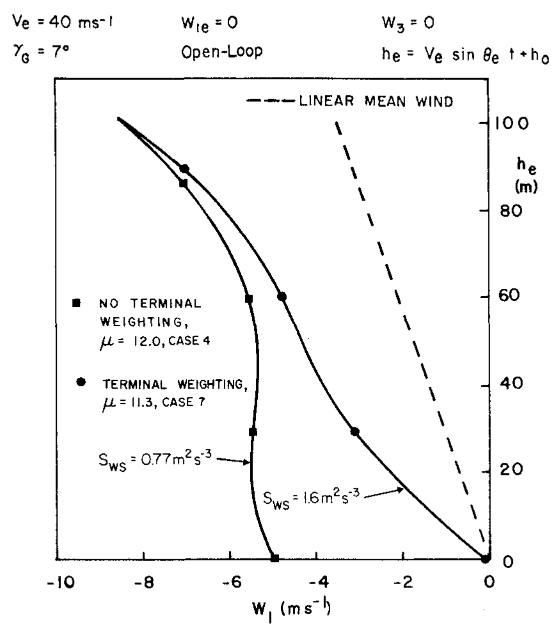

Fig. 5 Direct method example wind profiles. Effect of terminal weighting on $w_l(t_f)$ for small values of S_{ws}.

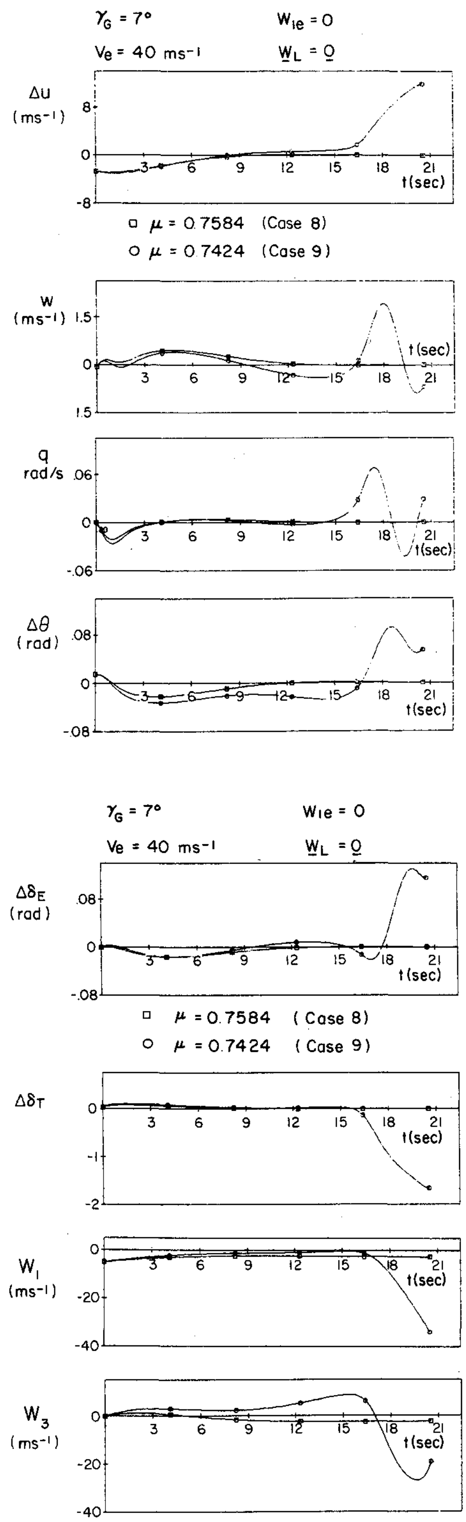

Fig. 6 DG method example.

weighting. For this example forcing $w_I(t_f)$ to be near zero was relatively straightforward; μ could be varied almost independently of the terminal weighting in order to achieve a desired S_{ws} value. This is not necessarily the case for all formulations with terminal wind velocity weightings, as was demonstrated for a differential game example presented in Ref. 9.

Differential Game Example

In this example a worst-case wind controller and a best-case aircraft controller are found that minimaximize the payoff functional (23) under the conditions (28a), (28b), (28e), (28f),

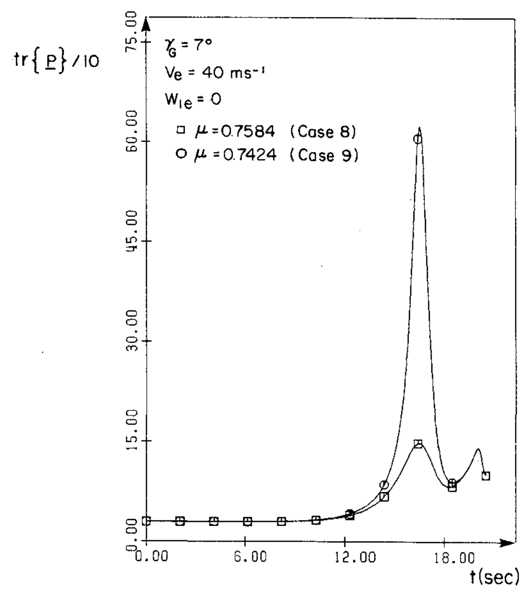

Fig. 7 *trP(t)* for two values of μ; DG method example.

(28g), and (28j). Also,

$$\Delta x_{op} = [\Delta x_I^T \Delta\delta^T]^T \tag{29a}$$

$$\Delta w_{op} = [w_I w_3]^T \tag{29b}$$

$$\Delta x_{op}(0) = [-5\ \mathrm{m\,s^{-1}}, 0]^T \tag{29c}$$

and

$$W_L = 0 \tag{29d}$$

Two cases will be discussed, as summarized in Table 2. Neither of these cases had weightings in the payoff functional on w_I ($\dot{w}_I$ was weighted). The resulting aircraft response, best-case control inputs and worst-case wind inputs are given in Fig. 6.

This formulation was found to encounter a conjugate point for $\mu < \mu_{cp}$ where $0.7384 < \mu_{cp} < 0.7424$. The minimax aircraft controller was found to be quite effective against the worst-case wind controller except for μ very near to μ_{cp}, where small changes in μ produced marked changes in controller performance. The worst-case wind solution sensitivity to μ variation was markedly greater than for the direct method example. The latter appears to be a consequence of the two-sided nature of the optimization, in which the wind controller can gain a significant advantage over the aircraft controller only for μ near the conjugate point. However, to obtain such good performance, the aircraft controller may require unrealistically large control deflections (see δ_T vs t curve of case 9 in Fig. 6). The latter suggests that if the control limiters had been included a significant degradation of aircraft controller performance would have resulted.

From Fig. 6 it is also apparent that case 9 exhibits strong wind disturbances for $t > 15$ s. This may be explained in terms of the matrix $P(t)$ that characterizes the minimax solution [see Eqs. (3a), (3b), and (4a)]. The trace of $P(t)$ has a peak near $t = 16$ s as is evident from Fig. 7. This peak is much greater for case 9.

Figure 7 suggests that for the values of μ chosen, this example has an infinite terminal time solution [i.e., a steady-state solution for $t_f \to \infty$ in the payoff functional (23]. This infinite terminal time solution was computationally shown to exist. The phugoid mode eigenvalues of the system for this infinite terminal time solution are summarized in Table 3. The wind controller alone leads to an unstable system, but the minimax solution itself is stable with considerably greater damping than the open-loop damping. This supports the previous observation that the aircraft controller is doing well against the worst-case wind controller.

The examples provide some insight into the type of results that may be obtained using optimal methods. While the wind models that are obtained have no direct connection with the physics of atmospheric flows, the optimization problems may be posed in a way so that certain wind constraints that are based on real wind conditions are satisfied. In these examples consideration has been given to an isoperimetric wind "energy" constraint S_{ws} and a terminal wind velocity condition.

Such worst-case wind models may prove to be useful in a number of applications. One of these is in the certification process (see Refs. 8 and 9). Another application that specifically takes advantage of the closed-loop nature of the worst-case wind inputs is that of flight simulator wind modeling. Because the wind inputs depend on the perturbed state of the aircraft, the winds to which the pilots are exposed will be different from run to run and pilot to pilot. A preliminary evaluation of the latter application has been carried out on the UTIAS multipurpose fixed-base simulation facility, with encouraging results.[17]

V. Summary and Recommendations for Future Work

The essence of the direct and differential game techniques lies in their conceptualization of the wind as an intelligent adversary that acts through a wind controller. This not only suggests worst-case behavior in a very appealing way, but also permits a diverse body of mathematical and conceptual tools to be applied to determining the worst-case wind models. In the examples optimal linear quadratic methods were employed in a number of formulations that illustrate the flexibility of the wind controller concept. The worst-case wind inputs were specified as a function of state, and thus, once this control law was determined, wind disturbances were available for all possible states. In the application, the wind inputs could be treated either as being determined by the state during the simulation or as time histories specified *a priori* for a particular set of initial conditions.

The perversity function method yields worst-case solutions in closed-loop form but is, for wind modeling purposes, *a programming method.* This is a consequence of the minimizing nature of the formulation in which the perversity enters the problem solely through the specification of a suitable perversity function rather than through a maximizing wind controller.

The recommendations for future work are as follows:

1) A thorough assessment of the usefulness of the worst-case wind controller models to flight simulator wind modeling.

2) A thorough assessment of the application of the worst-case functional maximization wind modeling methods (not necessarily wind controller methods) to certain aspects of autoland certification—this assessment requires consideration of nonlinear techniques and would ultimately require comparison and validation with existing methods (e.g., Jones' statistical discrete gust method[8] and Monte Carlo techniques); the application of these methods to other aspects of aircraft certification (e.g., structural certification), and indeed to other dynamic problems, may also prove to be fruitful.

3) An assessment of the validity of the linearized dynamic model for generating worst-case wind models, i.e., of the loss of optimality of the models when nonlinear dynamic effects are included.

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